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A NEW CLASS OF GRACEFULL TREES

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Abstract : The gracefulness of Tp-tree of size (5n, 5n-1) is obtained

Introduction:

Most graph labeling methods trace their origin to one introduced by Rosa [2] or one given Graham and Sloane [1]. Rosa defined a function f, a β-valuation of a graph with q edges if f is an injective map from the vertices of G to the set {0, 1, 2 ,…,q} such that when each edge xy is assigned the label |f(x)-f(y)|, the resulting edge labels are distinct.

A. Solairaju and K. Chitra [3] first introduced the concept of edge-odd graceful labeling of graphs, and edge-odd graceful graphs.

A. Solairaju and others [5,6,7,8,9] proved the results that(1) the Gracefulness of a spanning tree of the graph of Cartesian product of P_m and C_n,was obtained (2) the Gracefulness of a spanning tree of the graph of cartesian product of S_m and S_n, was obtained (3) edge-odd Gracefulness of a spanning tree of Cartesian product of P_2 and C_n was obtained (4) Even -edge Gracefulness of the Graphs was obtained (5) ladder $P_2 \times P_n$ is even-edge graceful, and (6) the even-edge gracefulness of $P_n \circ nC_5$ is obtained.

Section I : Preliminaries

Definition 1.1: Let G = (V,E) be a simple graph with p vertices and q edges.

A map f :V(G) → {0,1,2,…,q} is called a graceful labeling if

(i) f is one – to – one
(ii) The edges receive all the labels (numbers) from 1 to q where the label of an edge is the absolute value of the difference between the vertex labels at its ends.

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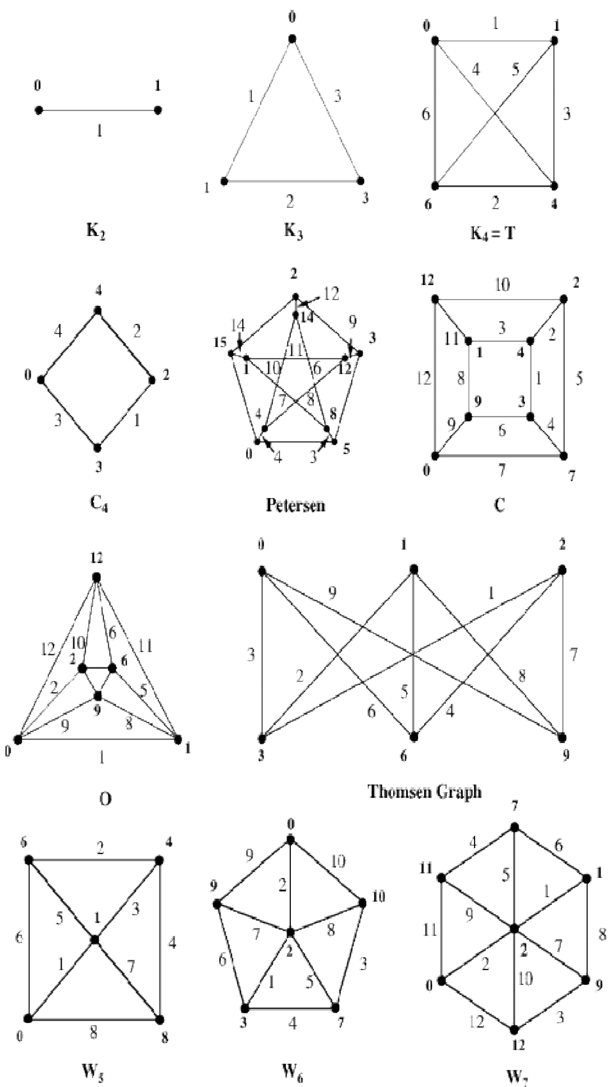
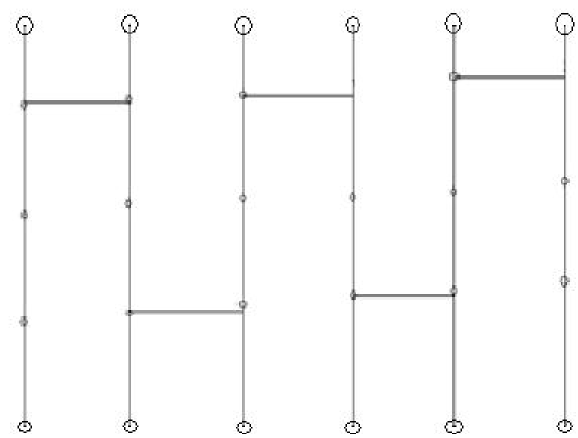
A graph having a graceful labeling is called a graceful graph.

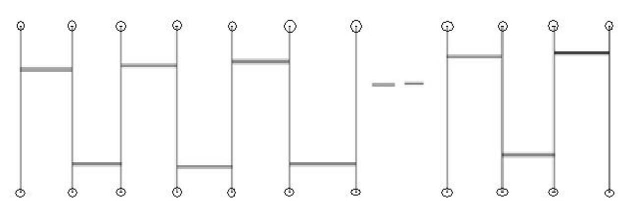
Example 1.1: The circuit C_4 is a graceful graph as follows:



Section – II: Tp–(5n, 5n-1) tree

Definition 2.1: n Δ P_5 is a tree, becoming a path by moving edges between vertices of degree 3 defined in the following manner only. That is, it is a T_p-tree obtained from n copies of P5, and connected acyclic in the following manner.





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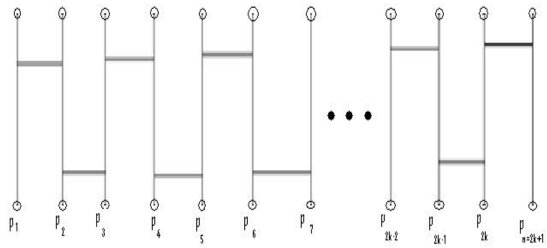


Figure 1 (n is odd)

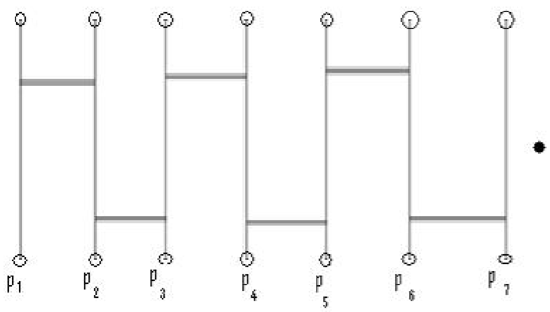


Figure 2 (n is even)

Main theorem 2.2: The connected graph Tp–tree with p=5n and q=5n-1 is graceful.

Proof: Due to definition (2.1), Tp–(5n,5n-1) is a connected graph (see figures 1 and 2) according as n is odd or even.

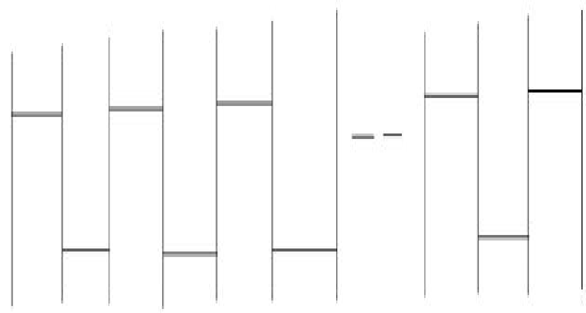
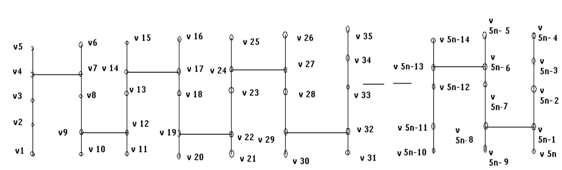


Figure 2 (n is even)

Case (1): n is odd.

The labelings of vertices and edges for Tp–(5n,5n-1) (Figure 1) are as follows:



Define f: v(G) → {0,1,2,…q)

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By

$$f(v_i) = \frac{i-1}{2} \quad ; \quad i=2,4,6\ldots5n$$

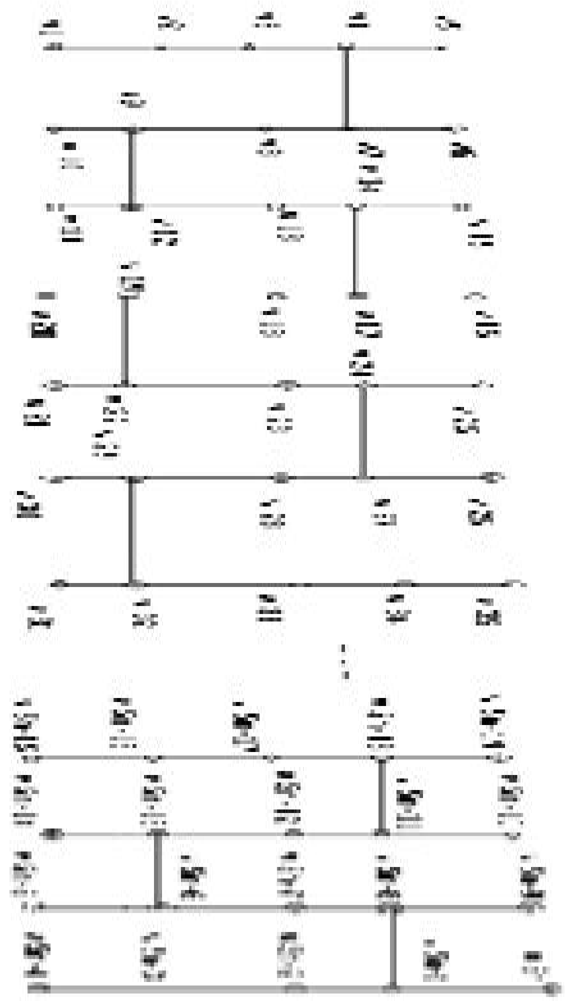
$$f(v_i) = q-\left(\frac{i-1}{2}\right) \quad ; \quad i=1,3,5\ldots5n-1$$

Define f_+:E(G)→ {1,2,3..q} by f_+ (u,v)= |f(u)-(v)|, $\forall$ u,v Є E(G).

Hence, the bisection maps f for vertices and f_+ for edges in Tp–(5n,5n-1) satisfies all condition of graceful labeling. Thus, Tp–(5n,5n-1) is a graceful if n is odd.

Case (2): n is even.

The labeling of vertices and edges for Tp–(5n,5n-1) (Figure 2) are as follows:



Define f: v(G) → {0,1,2,…q)

By

$$f(v_i) = \frac{i-1}{2} \quad ; \quad i=1,3,5\ldots5n-1$$

$$f(v_i) = q-\left(\frac{i-1}{2}\right) \quad ; \quad i=2,4,6\ldots5n$$

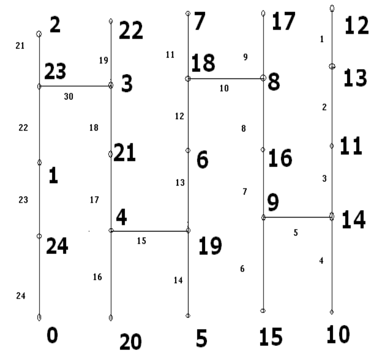
Define f_+: E(G)→ {1,2,3..q} by f_+ (u,v)= |f(u)-(v)|, ∀ u,v Є E(G).

Hence, the bisection maps f for vertices and f_+ for edges in Tp–(5n,5n-1) satisfies all the conditions of graceful labeling. Thus, Tp–(5n,5n-1) is a graceful if n is even.

Corollary 2.3: Tp–(5n,5n-1) is a graceful tree

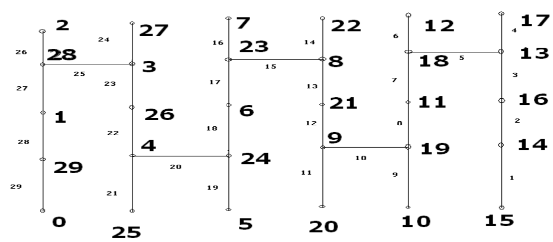
Example 2.1 :

The graph 5 Δ P_5 is a graceful graph.



Example 2.2:

The graph 6 Δ P_5 is a graceful graph.



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